# The molecular weight dependence of fatigue crack propagation in polycarbonate

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Fatigue crack propagation has been studied in polycarbonate as a function of specimen thickness and molecular weight. It was found useful to estimate the relative contribution of the shear lips and the plane strain crazing mode to fatigue crack propagation. In addition to measuring the width of the shear lips, the craze in the central region of the crack tip was examined in an optical microscope. The shear lip contribution was found to be particularly important at high values of the range of the stress intensity factor  $\Delta K$ , and was appreciable in all but the thickest specimens and the lowest values of  $\Delta K$ .

As in the case of fracture toughness, the fatigue crack propagation behaviour of polycarbonate is greatly affected by the molecular weight of the polymer. This is due to changes in both the shear lip contribution and the plane strain craze contribution. Because of the complicated nature of the failure mode it is suggested that the application of the Paris equation to the fatigue crack propagation of polycarbonate will be of limited significance.

#### 1. Introduction

The application of fracture mechanics to fatigue crack propagation in polymers is now well established and has been the subject of several notable reviews [1, 2]. It has been shown that a very useful starting point can be obtained by the application of the Paris equation [3]

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \alpha (\Delta K)^m \tag{1}$$

where  $a = \operatorname{crack}$  length,  $N = \operatorname{number}$  of cycles,  $\Delta K = \operatorname{range}$  of the stress intensity factor, and  $\alpha$ and m are constants, which may depend on the material and the test conditions. Over an intermediate range of values for  $\Delta K$ , the Paris equation has been shown to fit fatigue crack growth data for polycarbonate [2, 4]. It was found that the material was comparatively resistant to fatigue crack propagation, with no significant sensitivity to cyclic frequency or changes in cyclic wave form.

In a previous publication from this laboratory the influence of molecular weight on the cleavage fracture behaviour of polycarbonate was described [5]. In addition to direct measurements of fracture toughness and shear lip width, the shape of the craze at the crack tip was determined from the fringe pattern in reflected light. It proved possible to make a consistent interpretation of the results by assuming that the total strain energy release rate was comprised of a contribution associated with the craze at the centre of the crack tip and a shear lip contribution, both of which are affected by the molecular weight of the polymer.

In the present paper, this general approach is extended to fatigue crack propagation, and it will be shown to provide a satisfactory understanding of the influence of molecular weight and sample thickness.

#### 2. Experimental

The fatigue crack propagation tests were undertaken using a pneumatic uniaxial tensile fatigue tester, which was designed and built by Imperial Chemical Industries Ltd, Plastics Division, Welwyn Garden City. The apparatus, which has been described in detail previously [6, 7], provides a cyclic load in the form of a square wave with a

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Figure 1 Compact tension specimen.

rise time of about 50 msec. Throughout this investigation the load was varied between the desired maximum load and zero load at a frequency of 1 Hz. The fatigue tests were carried out on compact tension specimens with the geometry shown in Fig. 1. Specimens of three thicknesses, 3 mm, 6 mm and 9 mm were examined. These were cut from large sheets of polycarbonate and milled to the desired dimensions. To produce a sharp initial crack a razor blade was pushed slowly by hand into the root of the saw cut.

As in the previous study of fracture toughness and craze shape, a range of molecular weight materials was obtained by electron irradiation [8, 9]. The molecular weight characteristics of the materials studied are shown in Table I.

Experiments were performed at room temperature and  $-30^{\circ}$  C. The choice of  $-30^{\circ}$  C for detailed studies of fatigue crack growth and craze shape was suggested by the previous fracture studies, where it was shown to be a satisfactory temperature for stable crack propagation from a single craze. At room temperature there is a considerable possibility of multiple crazes at the crack tip, as also reported by other workers [10]. The low temperature tests were achieved by passing carbon dioxide from a cylinder at a controlled rate into the specimen test chamber. A transparent door on the test chamber allowed viewing of the specimen while under test and a thermocouple placed near to the specimen showed that the variation in temperature did not exceed  $\pm 2^{\circ}$  C once the equilibrium temperature had been reached. For each test at  $-30^{\circ}$  C a period of at least half an hour was allowed before the test was begun.

At slow crack speeds the crack growth was measured with a travelling microscope. At the highest crack speeds this was not possible and in these cases the crack speeds were taken from microscopic observation of the fatigue striations on the fracture surface. It has been established that at high crack speeds the crack speed corresponds to the distance between striations on the fracture surface [11]. At lower speeds this is not so [12].

## 3. Theory

**3.1.** Analysis of compact tension specimens Stress intensity factors for this type of specimen geometry have been calculated by Gross and Srawley [13]. They presented their results in terms of a dimensionless quantity

$$\frac{KBW^{\frac{1}{2}}}{P} = Y(a/W)$$
(2)

where K is the stress intensity factor, B is the specimen thickness, P is the applied load, a is the crack length and W the specimen width.

It can be shown that for the compact specimen used in this investigation

$$Y(a/W) = \left\{ \frac{-15.46}{[1-(a/W)]} + \frac{24.12}{[1-(a/W)]^2} + \frac{4.95}{[1-(a/W)]^3} \right\}^{\frac{1}{2}}$$
(3a)

for 
$$0.20 < (a/W) < 0.55$$

Material	$\bar{M}_{\mathbf{n}}$	${ar M}_{ m w}$	Test temperature
Unirradiated Lexan sheet	7150	18 000	Room temperature
	4250	13 000	and $-30$ C
	5500	14 500	Room temperature
Lexan sheet irradiated with 3 MeV electrons	6250	15 800	
	4900	13 800	
	5600	14 500	— 30° C
	6200	15 800	

TABLE I Molecular weight characteristics of polycarbonate polymers

and 
$$Y(a/W) = \left\{ \frac{43.19}{[1 - (a/W)]} - \frac{32.83}{[1 - (a/W)]^2} + \frac{18.62}{[1 - (a/W)]^3} \right\}^{\frac{1}{2}}$$
 (3b)

for 
$$0.55 < (a/W) < 0.70$$
 (see [5])

Thus the stress intensity factor can be calculated from a knowledge of the applied load and the specimen geometry. In the case of fatigue fracture the range of stress intensity factor is the important quantity, and this is generally written as  $\Delta K$ . Since in these experiments the minimum value of K is zero, then  $\Delta K$  is the same as the maximum applied stress intensity for any crack length.

#### 4. Results

# 4.1. Room temperature tests *4.4.1. Thickness effects*

Fig. 2 shows the room temperature fatigue crack propagation results for the unirradiated specimens of different thickness. It can be seen that the relationship between da/dN and  $\Delta K$  is approximately linear on the log-log plot over approximately an order of magnitude range in  $\Delta K$ . At high values of  $\Delta K$  the crack propagation rate rises more rapidly than this linear relationship would imply and at low values of  $\Delta K$  the crack propagation rate becomes zero so that there is a threshold value  $\Delta K_{\rm TH}$ . These results are similar to those observed previously for polycarbonate [2] and are typical of the behaviour of many polymers.

The results in Fig. 2 show that the 3 mm sheet had the greatest resistance to fatigue crack propagation, the 9 mm sheet giving a crack speed of an order of magnitude greater for any particular value of  $\Delta K$ , over most of the range tested, with the 6 mm sheet results in general falling between results for the other sheets. At lower values of  $\Delta K$ the data for the 6 mm sheets were essentially the same as those for the 9 mm sheet, whereas at high values of  $\Delta K$  the 6 mm sheet data were similar to those for the 3 mm sheet. It will be shown that these results can be understood in terms of different balances between craze formation and ductile shear lips for different values of thickness and  $\Delta K$ . At this stage we will only note that examination of the fracture surfaces showed that there is a complete range of different fracture modes from the propagation of a craze with comparatively narrow shear lips in the 9 mm sheets at low values of  $\Delta K$  to completely ductile



Figure 2 Effect of specimen thickness on fatigue crack propagation in unirradiated Lexan sheet ( $\overline{M}_n = 7150$ ).  $\triangle 9 \text{ mm}; \circ 6 \text{ mm}; \bullet 3 \text{ mm}.$ 

fracture in the 3 mm sheets at high  $\Delta K$ . Furthermore, as shown in Fig. 3, the shear lips increased in size as the crack propagation rate is increased at higher values of  $\Delta K$ . Following the philosophy of our previous paper, the width of the shear lip, in simple terms, depends on the relative value of the stress to cause yielding compared with that to cause crazing. Previous work indicates that the craze stress is more strain-rate dependent than the yield stress. The increase in shear lip width with increasing  $\Delta K$  can therefore be attributed to the increasing strain rate associated with the faster rise time of the machine at high  $\Delta K$  and possibly



Figure 3 Variation of shear lip width with  $\Delta K$  at room temperature for unirradiated Lexan sheet. A 9 mm; • 6 mm; • 3 mm.



Figure 4 Shear lip width as a function of crack speed at room temperature for unirradiated Lexan sheet. 
9 mm;  $\circ$  6 mm;  $\triangle$  3 mm.

the higher crack propagation rate. This point is discussed further below. Fig. 4 shows the variation of shear lip width with crack velocity, from which it can be noted that the difference between the specimens of different thickness is no longer so apparent.

#### 4.1.2. Molecular weight dependence

The molecular weight dependence of fatigue crack propagation is shown in Fig. 5 for the 3 mm thickness sheet. It can be seen that there is a very considerable increase in the resistance to fatigue crack propagation with increasing molecular weight over the whole range of the stress intensity factor. The results in Fig. 6 show that this is 638



Figure 5 Molecular weight dependence of fatigue crack propagation at room temperature in 3 mm thickness specimens.  $-\bar{M}_{n} = 7150; \quad \times \bar{M}_{n} = 6250; \quad \circ \bar{M}_{n} = 5500;$  $\Box \bar{M}_{n} = 4250; \quad + \bar{M}_{n} = 3700.$ 



Figure 6 Molecular weight dependence of shear lip width at different crack speeds in 3 mm thickness specimens at room temperature.  $\Box da/dN = 40 \,\mu m \, cy \, cle^{-1}$ ;  $\triangle da/dN =$  $10 \,\mu\mathrm{m}\,\mathrm{cycle}^{-1}$ ;  $\circ \,\mathrm{d}a/\mathrm{d}N = 1 \,\mu\mathrm{m}\,\mathrm{cycle}^{-1}$ .



Figure 7 Effect of specimen thickness on fatigue crack propagation of unirradiated Lexan sheet at  $-30^{\circ}$  C.  $\wedge$  9 mm;  $\bullet$  6 mm;  $\bullet$  3 mm.

clearly related to the increase in the width of the shear lips with increasing molecular weight. This is clearly very much in line with the results described previously for the cleavage fracture of this polymer.

## 4.2. Low temperature tests

## 4.2.1. Thickness effects

The fatigue crack propagation results obtained at  $-30^{\circ}$  C for unirradiated sheet of the three thicknesses are shown in Fig. 7. Compared with room temperature the dependence on specimen thickness was greatly reduced, the only significant thickness dependence occurring at high values of  $\Delta K$ , where the thinner specimens showed the greatest resistance to crack propagation. This can be attributed to the very much reduced widths of the shear lips at lower temperatures. It appears that the craze stress is less temperature dependent than the yield stress.

Viewing the results empirically in terms of the



Figure 8 Effect of molecular weight on fatigue crack propagation at  $-30^{\circ}$  C in 6 mm thickness specimens.  $\circ \overline{M}_n = 7150; \times \overline{M}_n = 6200; \bullet \overline{M}_n = 5600; \land \overline{M}_n = 4900.$ 

Paris equation, these low temperature results would give values of m three or four times higher than those at room temperature. This result is consistent with data obtained previously by Martin and Gerberich [14].

#### 4.4.2. Molecular weight dependence

Fig. 8 shows that there was a decrease in resistance to fatigue crack propagation as the molecular weight was decreased by increasing degrees of irradiation. As at room temperature this is clearly related to the molecular weight dependence of the shear lip width, which is shown in Fig. 9 for different crack speeds. It can also be seen from Fig. 8 that the value of  $\Delta K$  at which crack instability occurred decreased with decreasing molecular weight.

#### 5. Discussion

# 5.1. Mixed mode fatigue crack propagation $at - 30^{\circ} C$

The results described above suggest that there are links between this study of fatigue crack propagation and the previous investigation of fracture toughness. We will therefore discuss the  $-30^{\circ}$  C fatigue results in the first instance and establish the validity of this proposed link.



Figure 9 Molecular weight dependence of shear lip width  $\omega_f$  for various crack speeds at  $-30^\circ$  C.  $\Box da/dN = 400 \,\mu$ m cycle<sup>-1</sup>;  $\triangle da/dN = 100 \,\mu$ m cycle<sup>-1</sup>;  $+ da/dN = 10 \,\mu$ m cycle<sup>-1</sup>.

It was shown that the cleavage fracture results could be described by an equation of the form

$$G_{\rm CM} = G_{\rm IC} \frac{B-\omega}{B} - \phi \frac{\omega^2}{2B}$$

where  $G_{CM}$  is the overall strain energy release rate,  $G_{IC}$  is the plane strain energy release rate (associated with the craze),  $\omega$  is the total width of the shear lips evident on the fracture surface, and  $\phi$  is the energy associated with deforming to failure to unit volume of material in the shear lips.

In the case of fatigue crack propagation, examination of the situation at the crack tip and the fracture surfaces, indicated that very similar mixed-mode failure occurred. It is therefore proposed that the fatigue failure can be described by a similar equation so that

$$G_{\mathbf{CM}}^{\mathbf{f}} = G_{\mathbf{IC}}^{\mathbf{f}} \frac{B - \omega_{\mathbf{f}}}{B} + \phi_{\mathbf{f}} \frac{\omega_{\mathbf{f}}^2}{2B}$$

where  $G_{CM}^{f}$  is the overall strain energy release rate, corresponding to  $\Delta K$ ,  $G_{IC}^{f}$  is the strain energy release rate associated with the crazing mode of failure,  $\omega_{f}$  is the total width of the shear lips and  $\phi_{f}$  is the energy associated with deforming to failure unit volume of material in the shear lips.

The fatigue tests were stopped when the crack growth rate had been determined and a small section surrounding the crack tip carefully cut out, polished on a buffing wheel and examined in an optical microscope in reflected light [5-7]. From the interference pattern the craze profile in the relaxed state was determined. Fig. 10 shows a typical interference pattern and a typical craze profile is shown in Fig. 11. As in the previous work on fracture [15, 16], the craze profile compares well with the shape predicted by Rice [17]



Figure 10 A typical interference fringe pattern from the craze at a crack tip in a fatigued specimen.



Figure 11 Comparison of the experimental craze profile with that predicted by the Rice formulation of the Dugdale zone model for the craze at a crack tip.  $\triangle$  observed from  $-30^{\circ}$  C fatigue crack propagation in unirradiated polycarbonate: —— predicted.

for the Dugdale plastic zone model. This was true for all levels of  $\Delta K$ . We shall therefore proceed, as in our previous studies of fatigue crack propagation in vinyl urethane polymers [6, 7], on the basis that at any level of  $\Delta K$ , the craze size is such as to exactly cancel the stress singularity in the plane strain region of the crack for the maximum value of the applied stress intensity factor  $K_{\rm IC}^{\rm f}$ , where  $(K_{\rm IC}^{\rm f})^2 = G_{\rm IC}^{\rm f}(E/1 - \nu^2)$ . (E =Young's modulus and  $\nu =$  Poisson's ratio).

From Rice's formulation of the Dugdale zone model the crack opening displacement  $\delta_t^f$  is given by

$$\delta_{t}^{f} = \frac{G_{IC}^{i}}{\sigma_{c}}$$

where  $\sigma_c$  is the craze stress, and the craze length  $r_c^f$  is given by  $-(v_c^f)^2$ 

$$r_{\rm c}^{\rm f} = \frac{\pi}{8} \frac{(K_{\rm IC}^{\rm i})^2}{\sigma_{\rm c}}$$

The optical measurements determine  $\delta_t^f$  and  $r_c^f$  from which  $G_{IC}^f$  and  $\sigma_c$  can be derived, assuming a value of  $E/(1-\nu^2)$  from the previous studies of cleavage fracture. Since only the relaxed craze profile was determined it was necessary to assume a value for the extension of the craze, and this was taken to be 1.65 i.e. identical to that found from the cleavage fracture.

The results for unirradiated 3 mm sheet are summarized in Table II, and those for sheet of similar thickness but irradiated to reduce the molecular weight very appreciably in Table III. There are several interesting points to note regarding these results. First, the craze stresses for the lower molecular weight material are significantly lower than for the higher molecular weight material, similar to the previous findings for cleavage fracture. The craze stress for each material increases as  $\Delta K$  is increased and the crack propagation rate increases. This can be attributed to the increase in craze stress with strain rate. At low crack speeds the strain rate in the craze at the crack tip will depend primarily on the loading rate which is determined by the pneumatic response of the machine. As the machine was designed to give an approximately constant rise time, the strain rate is increased when  $\Delta K$  is increased. At higher crack propagation rates it is possible that the crack speed itself will influence the rate of strain of the material within the craze, so that the craze stress will be further increased. As noted before, the craze stress is more dependent on strain rate than on the yield stress, so that the increase in craze stress with increasing strain rate gives rise to increases in shear lip width.

Secondly, the different levels of craze stress for the different levels of molecular weight also explain the much increased shear lip widths in the higher molecular weight material. The contribution of the shear lips to the total strain energy release rate varies very much indeed, from a small absolute contribution of 0.078 kJ m<sup>-2</sup> in the low molecular weight sample at low  $\Delta K$  to 0.69 kJ m<sup>-2</sup> in the high molecular weight sample at high  $\Delta K$ . It is important to note that the value for  $\phi_{f}$  is very nearly constant, and in particular does not differ significantly for the samples of different molecular weight. This is consistent with the previous study of cleavage fracture and moreover the values of  $\phi_{\rm f}$  are very close to that of 120 MJ m<sup>-3</sup> estimated for cleavage fracture behaviour.

# 5.2. The importance of polymer molecular weight in fatigue crack propagation

The importance of molecular weight on the fatigue crack propagation has been shown in Fig. 5 for room temperature results and Fig. 8 for the  $-30^{\circ}$  C results. A more illustrative plot is shown in Fig. 12 for the room temperature results. Here, values of  $\Delta K$  which would give crack speeds of 0.2, 1, 4 and  $20 \,\mu\text{m}$  cycle<sup>-1</sup> are plotted as a function of  $\overline{M}_n$  and  $\overline{M}_w$ . Extrapolation of the data suggests that even very low values of  $\Delta K$  would give very fast crack speeds for molecular weights corresponding to  $\overline{M}_n = 3500$  and  $\overline{M}_w = 11500$ . In other words, the resistance to fatigue crack growth approaches zero at this level of molecular weight. This result

ΔK (MN m <sup>-3/2</sup> )	Maximum strain energy release	$\operatorname{Craze}_{r_G^{\mathbf{f}}}$	Crack opening displacement	Craze stress $\sigma_{c}$	Plane strain strain energy	Shear lip contributio	п	(سا 1 س <sup>- ع</sup> ) مور	Crack speed (µm cycle <sup>-1</sup> )
	rate, $G_{CM}^{f}$ (kJ m <sup>-2</sup> )	(m <sup>#</sup> )	δ <sup>1</sup> (μm)	(MN m <sup>-2</sup> )	release rate $G_{\rm IC}^{\rm f}$ (kJ m <sup>-2</sup> )	(kJ m <sup>-2</sup> )	$\%$ of $G^{\mathbf{f}}_{\mathbf{CM}}$		
1.95	1.33	100	10.0	112	1.12	0.21	16.2	127	2.0
2.15	1.62	103	10.9	118	1.28	0.34	20.7	116	6.0
2.30	1.86	93	10.6	128	1.36	0.50	26.9	119	13.0
2.48	2.16	94	11.1	132	1.47	0.69	31.8	130	30.0

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V m <sup>-3/2</sup> )	Maximum strain energy release	Craze length r <sub>c</sub>	Crack opening displacement	Craze stress o <sub>c</sub>	Plane strain strain energy	Shear lip contributio	ď	φf (MJ m <sup>-3</sup> )	Crack speed $(\mu m \text{ cycle}^{-1})$
	rate, G <sup>f</sup> CM (kJ m <sup>-2</sup> )	(m <sup>7</sup> )	$\delta \frac{f}{c}$ ( $\mu m$ )	(MN m <sup>-2</sup> )	release rate GIC (kJ m <sup>-2</sup> )	$(kJ m^{-2})$	% of G an		•
5	0.317	42	3.0	80	0.239	0.078	24.6	122	2.8
0	0.351	47	3.4	80	0.271	0.080	22.8	111	3.6
8	0.489	62	4.6	82	0.375	0.114	23.3	122	6.0
0	0.593	64	5.0	88	0.442	0.151	25.5	112	10.0



Figure 12 Molecular weight dependence of the  $\Delta K$  required to give various crack speeds at room temperature in 3 mm thickness specimens of polycarbonate.  $+ da/dN = 20 \,\mu\text{m}$ cycle<sup>-1</sup>;  $\nabla da/dN = 4 \,\mu\text{m}$  cycle<sup>-1</sup>;  $\Box da/dN = 1 \,\mu\text{m}$  cycle<sup>-1</sup>;  $\circ da/dN = 0.2 \,\mu\text{m}$  cycle<sup>-1</sup>.

agrees well with those obtained in cleavage fracture which showed that the craze dimensions became very small and the craze stress had a very low value in this range of molecular weight.

It can be noted from Figs. 6 and 9 that the width of the shear lips tends to zero for a molecular weight level corresponding to  $\overline{M}_{n} \sim 4000$  and  $\overline{M}_{w} \sim 12250$  which is slightly higher than the level at which the fatigue crack resistance tends to zero. There is therefore a small range of polymer molecular weight for which the fracture surfaces show no shear lips.

It is of some interest to speculate further on the role of molecular weight in determining fatigue crack propagation. Clearly one important factor is the change in balance between the plane strain craze contribution and the shear lip contribution to mixed mode failure. From the cleavage fracture studies, and from the attempts to analyse these fatigue data along similar lines, it is clear that the shear lip contribution is a major factor. Some theories of fatigue crack propagation suggest that the crack growth rate is proportional to the length of the plastic zone preceding the crack [18]. In polycarbonate, the lower the molecular weight, the lower is the craze stress at any particular strain rate, which means that a longer craze is required to cancel the same stress singularity at the crack tip. Thus if the crack growth increase is proportional to the craze length, then it would be expected that the lower molecular weight materials would give faster crack speeds. The situation is, however, so much complicated by the very large shear lip contribution, that no direct link exists between craze length and crack speed.

On a more fundamental level the influence of molecular weight on the craze stress suggests that as the polymer molecular weight is decreased, there are an increasing number of chains which fill space rather than contribute to the load-bearing network. Indeed it appears that there is a critical molecular weight level at which a load-bearing network ceases to exist, presumably due to the reduced number of entanglements.

## 5.3. The importance of the shear lips for room temperature fatigue crack propagation

As already noted the shear lip widths were considerably greater at room temperature than at  $-30^{\circ}$  C, which we have attributed to the greater temperature dependence of the craze stress rather than to the yield stress. The importance of the shear lip contribution at room temperature is brought out by considering the high values of  $\Delta K$  which were applied without causing acceleration and failure. The value of the plane strain (craze) fracture toughness  $K_{1C}$ , obtained from cleavage fracture data at room temperature was between 2 and 3 MN m<sup>-3/2</sup>, whereas the values of  $\Delta K$  were 6 MN m<sup>-3/2</sup> or more in the 3 mm thickness sheets. Only at extremely low values of  $\Delta K$ were the shear lips thin enough for the plane strain fracture mode to dominate.

The shear lips also influence the shape of the da/dN against  $\Delta K$  curves. At low crack speeds the shear lips were thin so that the thickness dependence of crack speed is likely to be small

in all but the thinnest sheets. Thus we would expect the curves for all thicknesses to converge as the crack speed tends to zero giving the same value for  $\Delta K_{TH}$ . This expectation is confirmed by the data shown here in Fig. 2. As  $\Delta K$  was increased, so did the crack speed and the shear lip width. Indeed the 3 mm thick specimens were completely ductile at high values of  $\Delta K$ . The 9 mm specimens showed a lower resistance to fatigue crack propagation than the 3 mm specimen because the relative contribution of the shear lips was smaller, and the state of stress was closer to plane strain.

The results for the 6 mm specimens are of particular interest because many previous studies have used material of about this thickness. At low values of  $\Delta K$  the results were virtually the same as those for the 9 mm specimens, but with increasing shear lip width at higher crack speeds the results became much closer to those for the 3 mm specimens. In other words, there was a change from conditions approaching plane strain to conditions approaching plane stress with increasing  $\Delta K$ . This observation is of great significance in the fatigue crack propagation testing of polycarbonate because it is usual to attach significance to the values of  $\alpha$ and *m* calculated from Equation 1. However, these results show that, due to the change from plane strain to plane stress,  $\alpha$  and *m* may not be directly identified as material parameters, even if the test temperature and frequency are fixed. Because the plane stress resistance to fatigue crack propagation is greater than the plane strain resistance, an equation of the form of Equation 1 suggesting a linear correlation between da/dN and  $(\Delta K)^m$ would be an inadequate description of fatigue data, even if the effects of mean stress, the instability at high  $\Delta K$  and the threshold  $\Delta K_{TH}$  can be neglected for a reasonable range of  $\Delta K$ .

The insensitivity of polycarbonate to test frequency has been discussed previously by Hertzberg *et al.* [19]. Radon and Culver [20] have also done tests over a wide temperature range and found that there is some frequency sensitivity at low temperatures. It is possible that at room temperature any frequency effect of the crazing mode is being swamped by the contribution of the ductile mode, while at lower temperatures this does not occur to the same extent. Although the yield stress of polycarbonate shows some dependence on strain rate, this is much less than that shown by the craze stress which may go some way towards explaining the lack of frequency sensitivity at room temperature compared with that of more brittle polymers, where failure is dominated by the failure of the craze.

## 6. Conclusions

(1) At room temperature there was a significant effect of sheet thickness on the fatigue crack propagation in polycarbonate, thicker specimens having a significantly lower resistance to fatigue crack propagation particularly at higher values of  $\Delta K$ . This thickness dependence was due to the influence of ductile shear lips.

(2) There was a substantial decrease in the resistance to fatigue crack propagation as the molecular weight of the polymer was decreased. Unstable fracture occurred at lower values of  $\Delta K$  with decreasing molecular weight.

(3) At  $-30^{\circ}$  C an equation of the form

$$G_{\rm CM}^{\rm f} = G_{\rm IC}^{\rm f} \frac{B - \omega_{\rm f}}{B} + \phi_{\rm f} \frac{\omega_{\rm f}^2}{2B}$$

was of use in estimating the relative contribution of the shear lips and the plane strain crazing mode to fatigue crack propagation. The shear lip contribution was found to be particularly important at high values of  $\Delta K$ .

(4) At room temperature the contribution of the shear lips was very considerable in all but the thickest specimens and lowest values of  $\Delta K$ . The significance of the values of  $\alpha$  and *m* obtain from the Paris equation

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \alpha (\Delta K)^m$$

is limited due to changes in the fracture mode with changes in thickness and  $\Delta K$ .

(5) It is likely that further improvements in fatigue crack propagation resistance for polycarbonate could be obtained by increasing the molecular weight of the polymer beyond the highest value examined in this investigation.

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